Polarized light in quantum dot qubit under an applied external magnetic field

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Single self-assembled quantum dot (SAQD) dependence on an external magnetic field has been theoretically and numerically studied. The carrier wave functions and energy eigenvalues have been evaluated and the differences between small and large quantum dots have been explored. The current investigation has shown the dependence of circular light polarization on an external magnetic field and on the size of the SAQD as well. Although the light polarization in the case of spin-polarized states along the direction [110] for large SAQD decreases by increasing the magnetic field, in the case of small dots the polarization increases.

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I. INTRODUCTION

Quantum information technology has been in the center of world wide research interests due to the desired quantum computer (QC) architecture.[1] The solid-state low-dimensional structures are devices promising to take a leader role for designing the QC. One of the possible material structures could be the quantum dots (QDs).[2,3] In recent years, two-dimensional (2D) and three-dimensional QDs have been excessively studied. More precisely, the confinement of carriers, spins in space and their excitations have been theoretically and experimentally explored. In the last few years, many interesting problems have been studied in 2D QDs and single self-assembled quantum dot (SAQD) structures. The problem of two electrons in horizontally coupled quantum dots (CQDs) has been explored including the electron spin.[4] This system could provide the necessary two-qubit entanglement required for quantum computation. Another feature of two electrons in CQDs under the existence of an external magnetic field has been studied and the fact that the system consists of composite particles has been shown.[5] Furthermore, the many-electron problem in fabricated rectangular-shaped QDs has been studied using different approaches, e.g., variational Monte Carlo and spin density-functional theory and has shown that the electronic structure is very sensitive to the shape of QD.[6] Charge decoherence in qubits was explored by using the Fermi golden rule via the emission of acoustic phonon emission.[3] Light polarizarion and more specifically circular polarization dependence of dipole recombination on spin-polarized states within a self-assembled quantum dot has been experimentally[7] and theoretically studied.[8] The above mentioned properties of QDs among others could be of special importance in quantum computer architecture in order to create quantum bits (qubits).

In this Brief Report, the spin-polarized states have been evaluated and their dependence on the size of the QD, external magnetic field and dot energy gap (E_g) has been shown. Furthermore, circular polarization dependence of dipole recombination on spin-polarized states within a self-assembled QD in the presence of an external magnetic field has been explored. The investigation has shown that in the case of large QDs, light polarization along the direction [110] (for elongation different than zero) decreases as B increases. On the other hand, light polarization in small QDs has opposite behavior.

II. THEORY

The strain dependent $\mathbf{k}\cdot\mathbf{p}$ theory has been employed to calculate the single electron and hole wave functions for an ellipsoidal cap QD made with InAs, as illustrated in Fig. 1. The QD is embedded on a wetting layer of InAs (with thickness 0.3 nm) and is surrounded by GaAs. The height of the QD has been fixed to $h=2.1$ nm and the width-to-height ratio $\{b=[d_{(110)}+d_{(1-10)}]/h\}$ and elongation $(e=d_{[110]}/d_{[1-10]})$ varied. The electron/hole wave functions were numerically computed on a real space grid with spacing equal to the wetting layer thickness. Strain and carrier confinement split the heavy hole and light hole degeneracy and the states which are denoted by $|\psi\rangle$ and $T|\psi\rangle$ (time reverses of each other) are doubly degenerate. The energy gap (E_g) of the SAQD structure strongly depends on the size of the dot as it is shown in our investigation. The material parameters which are involved in the $\mathbf{k}\cdot\mathbf{p}$ simulations are taken by Refs. 9 and 10. We construct the spin-polarized ground and first excited states for both electrons and holes by taking a linear combination of the states comprising the doublet and by adjusting the coefficient in order to maximize the expectation value of the pseudospin operator projected onto a direction l.[11] The requested complex number α maximizes the quantity

$$\frac{[\langle\psi| + \alpha^*\langle\psi|T]\hat{l}\cdot\mathbf{S}[|\psi\rangle + \alpha T|\psi\rangle]}{1+|\alpha|^2}, \tag{1}$$

where $\mathbf{S}$ is the pseudospin operator[11] in the eight-band $\mathbf{k}\cdot\mathbf{p}$ theory and l is the spin orientation.

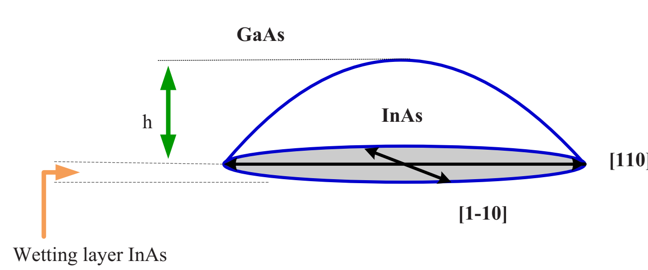

FIG. 1. (Color online) The geometry of the quantum dot made with InAs/GaAs and a fixed wetting-layer thickness to 0.3 nm.

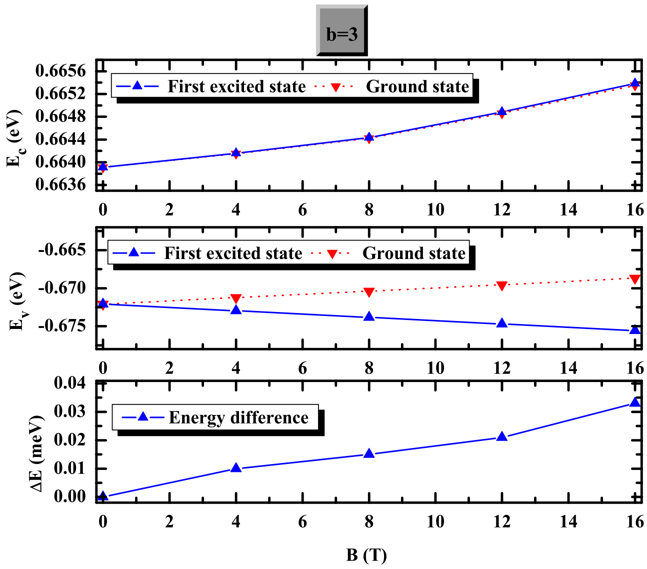

FIG. 2. (Color online) The electron-hole energy for the ground and first excited states and the energy difference between the two lowest conduction-band states for the case $b=3$ and $e=1.4$.

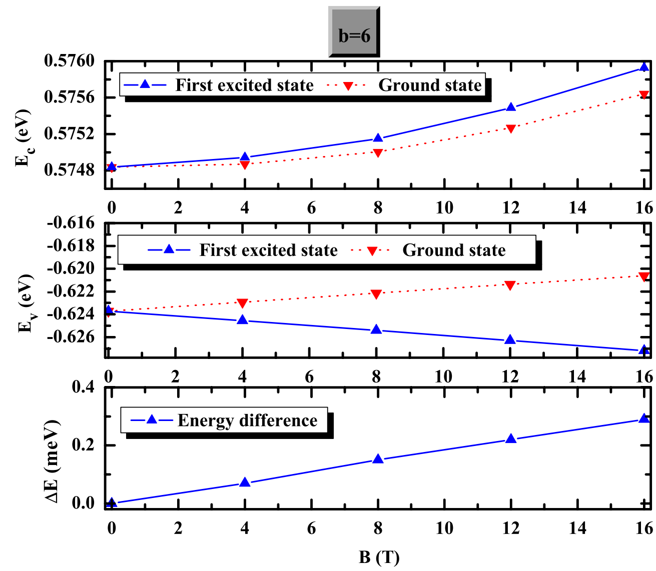

FIG. 3. (Color online) The electron-hole energy for the ground and first excited states and the energy difference between the two lowest conduction-band states for the case $b=6$ and $e=1.4$.

We are interested in the situation in which the electron spin is polarized along the same direction l as is the observed emitted light. The light polarization is given by the following:[12]

$$P_l = \frac{I_l^{(+)} - I_l^{(-)}}{I_l^{(+)} + I_l^{(-)}}. \qquad (2)$$

The symbol $I_l^{(\pm)}$ refers to the light intensity with $\pm$ helicity. The circularly polarized light intensity for spin-polarized electron and unpolarized holes could be given by

$$I_l^{(\pm)} = |\langle \psi_h | \hat{\epsilon}_l^{(\pm)} \cdot \mathbf{p} | \psi_e \rangle|^2 + |\langle \psi_h | T \hat{\epsilon}_l^{(\pm)} \cdot \mathbf{p} | \psi_e \rangle|^2 \qquad (3)$$

and for spin-polarized holes and unpolarized electron the intensity of circularly polarized light is given by

$$I_l^{(\pm)} = |\langle \psi_h | \hat{\epsilon}_l^{(\pm)} \cdot \mathbf{p} | \psi_e \rangle|^2 + |\langle \psi_h | \hat{\epsilon}_l^{(\pm)} \cdot \mathbf{p} T | \psi_e \rangle|^2. \qquad (4)$$

The indices e and h correspond to electron and holes, respectively. The momentum operator is noted as $\mathbf{p}$ and $\hat{\epsilon}_l^{(\pm)}$ is the circular polarization vector with helicity $\pm$. The above mentioned two different cases for the $I_l^{(\pm)}$ give identical results as a sequel of the anticommutation relations between $\mathbf{p}$ and T. This results in the emitted light polarization being independent from whether the injected spin-polarized carriers are electrons or holes.

III. RESULTS

The first part of the current investigation is the calculation of both the carrier energy and the dot energy gap. Figure 2 presents the electron (E_c) and the hole (E_v) energy for the case of small size QD ($b=3$) as a function of the applied magnetic field. The energy difference between the two lowest conduction-band states lies between 0 and 0.035 meV for the magnetic field range. The variation in the external magnetic field releases the dependence on the Zeeman energy. As the magnetic field increases the energy splitting increases due to the Zeeman energy splitting ($E_z = g\mu_B B$). Figures 3 and 4 illustrate the same information with $b=6$ and 10 (large QDs), respectively. Comparison with Figs. 2–4 shows that the energy splitting increases as the width-to-height ratio becomes larger. This feature could be explained by the dependence of E_z on Lande factor. The g factor increases as the size of the QDs becomes larger[13] which results in larger energy splitting by increasing the width-to-height ratio. Figure 5 presents the E_g as a function of the magnetic field for smaller and larger dots. Energy gap depends strongly on the size of the dot and less on the external magnetic field. For

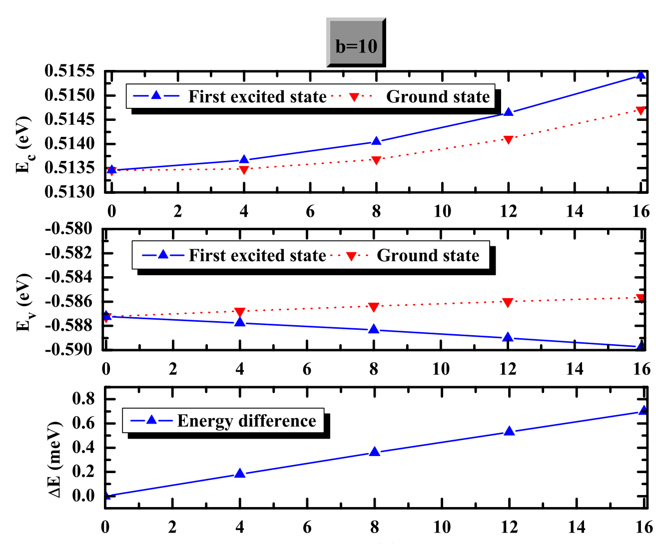

FIG. 4. (Color online) The electron-hole energy for the ground and first excited states and the energy difference between the two lowest conduction-band states for the case $b=10$ and $e=1.4$.

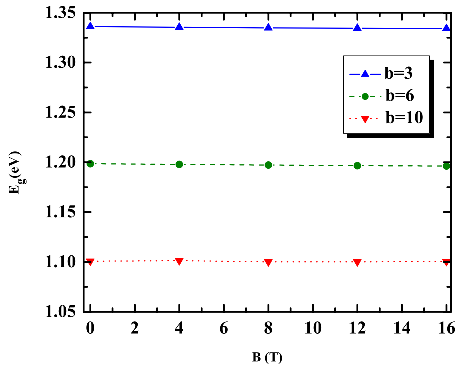

FIG. 5. (Color online) The energy gap (E_g) as a function of the applied magnetic field for different width-to-height ratio and $e=1.4$.

instance, in the case of $b=3$ and $B=0$ T ($B=16$ T) the value energy gap gets the value $E_g=1.336$ eV ($E_g=1.334$ eV), which brings the conclusion that the magnetic field slightly changes the energy gap of QD.

The conduction-band electron, for smaller dots, has larger energy than in the case of larger dots which results in larger energy difference between the carrier in bottom of the conduction and the top valence-band state (E_g). In the case of large QDs, the number of the bound states increases and the ground-state energy becomes smaller, which results in a smaller energy gap E_g.

A complete picture of the light polarization is presented in Fig. 6. As it is obvious, in small/large dots, polarization increases/decreases by increasing the magnetic field. This kind of behavior is due to the energy gap's dependence on the size of QD and due to the Zeeman splitting on the external magnetic field as earlier analyzed (Figs. 2 and 4). As earlier mentioned, the energy gap of small dots is larger than that of large dots. Furthermore, the carrier wave functions involved in Eq. (2) depend on the size of the QDSs (in other

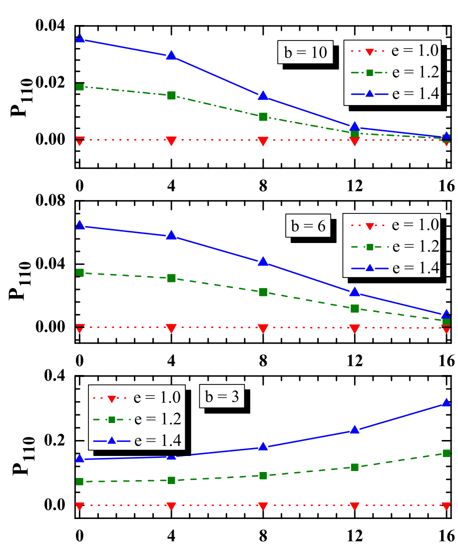

FIG. 6. (Color online) The polarization P_{110} as a function of the applied magnetic field for different elongation and width-to-height ratio with fixed dot height $h=2.1$ nm.

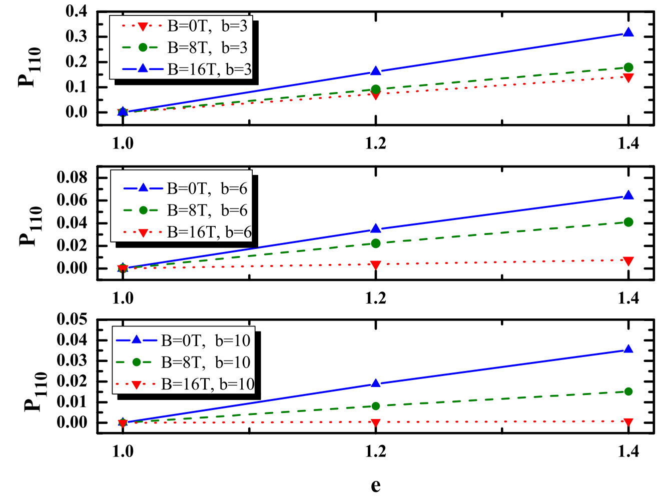

FIG. 7. (Color online) The polarization P_{110} as a function of the elongation for different applied magnetic fields and width-to-height ratio with fixed dot height $h=2.1$ nm.

words on the energy gap) and result in larger light polarization for the case of small dots (large E_g). Increasing the magnetic field in small QDs, the Eq. (2) results in larger values for the light polarization as a consequence of carrier wave-functions dependence on the magnetic field. It is worth mentioning that although the carriers are 100% polarized, the emitted light is less than 100% polarized. For example, in the case of $B=0$ T and elongation $e=1.4$ the results are consistent[14] with the numerical calculations[11] and the experimental results[7] (~5% circularly polarized light for 100% polarized carriers) previously reported. Increasing the E_g, it is possible for small QDs to increase the percentage of po-

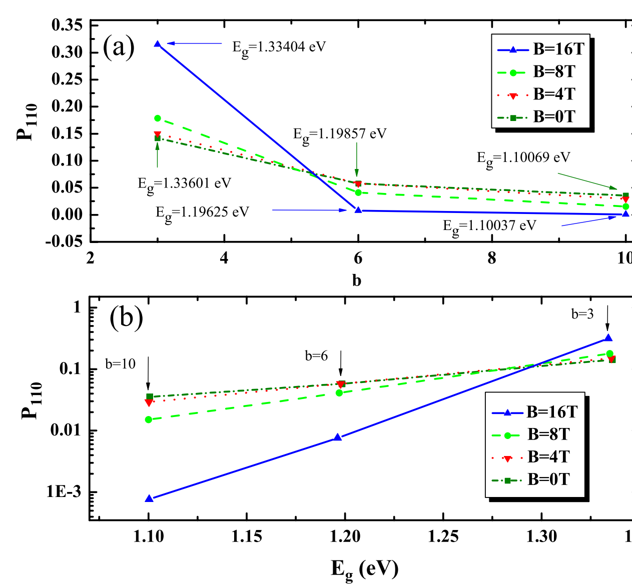

FIG. 8. (Color online) (a) The polarization P_{110} as a function of the width-to-height ratio (QD size) for different applied magnetic fields with fixed dot height $h=2.1$ nm and elongation $e=1.4$. (b) The polarization P_{110} as a function of the energy gap for different applied magnetic fields with fixed dot height $h=2.1$ nm and elongation $e=1.4$.

larized light by increasing the external magnetic field. In this case, for $B=0$ T and $e=1.4$ the light is $\sim 18\%$ polarized and for $B=16$ T, $e=1.4$ the light is $\sim 32\%$ polarized.

Elongation is an important parameter for controlling the light polarization. As it is shown for axial symmetric QD ($e=1.0$), light polarization vanishes [as it is obvious in Eq. (2)] due to the same value of light intensity for $\pm$ helicity. Increasing the elongation (Fig. 7), circular light polarization increases because of the azimuthal symmetry breaking. For the smallest QD the polarization receives the largest value ($P_{110} \sim 0.32$, for $e=1.4$, $b=3$, and $B=16$ T), as earlier explained.

An experimentally useful dependence of circular polarization is given in Fig. 8(a). As it is obvious, for fixed width-to-height ratio $b=3$, the smallest value of P_{110} is achieved for the case of $B=0$ T and the largest for $B=16$ T, although the energy gap slightly differs. Figure 8(b) presents the dependence of circular light polarization on the QD energy gap. This kind of behavior is reasonable as explained above.

In the case of polarization directions other than [110], the system is characterized by a completely different picture. For instance, the case in which the polarization direction is [001], the P_{001} gets a value close to 1 and it does not depend on the external magnetic field. For other polarization directions such as [100] and [010], the circular polarization vanishes $P_{100}=P_{010}=0$ for any of our QD configurations. So P_{001} is the best choice if one is interested in the light polarization along the growth direction.

IV. CONCLUSIONS

Summing overall, a suitable solid-state structure for quantum computing architecture has been studied and new features of the system in applicable conditions are searched. On one hand, the dependence of the energy gap on the size of the dot and on the other hand the variance of the energy splitting by increasing the external magnetic field, result in a very interesting feature of the circular light polarization. In this Brief Report, it has been shown that both the magnetic field and the geometry of the QD are parameters of special importance in order to control the light polarization along the direction of spin polarization in QD qubits. In the case of very small dots, circular light polarization along the direction [110] increases by increasing the magnetic field and in the case of large dots it decreases as the field increases. Although the carriers are 100% polarized, the emitted light is less than 100% polarized. Other polarization directions such as [001] and [100] lead to circular light polarization values 1 and 0, respectively. The above mentioned properties could be used in order to control the output of a qubit made with SAQDs.

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